Trigonometric Identity III

$$\cot^2\theta \equiv \operatorname{cosec}^2\theta - 1$$

This is a helpful equation used to relate the functions cotangent (otherwise known as cot) and cosecant (otherwise known as cosec). $\cot^2\theta$ is the same thing as $(\cot\theta)^2$, it is merely an easier way of writing it, the same is true for $\operatorname{cosec}^2\theta$. The ≡ symbol means "identical to" (i.e. cot squared theta is identical to cosec squared theta minus one). This symbols means the relationship is always true, regardless of the value of θ. θ is a placeholder for an angle, and for this identity to work the angle must be the same for both cot and cosec.

Proof

Starting with Trigonometric Identity I,

$$\sin^2\theta + \cos^2\theta \equiv 1$$

Dividing both sides by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} \equiv \frac{1}{\sin^2\theta}$$

Using Trigonometric Identity II, $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{1}{\tan^2\theta} \equiv \frac{1}{\sin^2\theta}$$

Simplifying $\frac{\sin^2\theta}{\sin^2\theta}$

$$1 + \frac{1}{\tan^2\theta} \equiv \frac{1}{\sin^2\theta}$$

Using our knowledge that $\cot\theta = \frac{1}{\tan\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$$1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$$

By re-arranging we find that

$$\cot^2\theta \equiv \operatorname{cosec}^2\theta - 1$$

See also

- Cosecant, Secant and Cotangent
- Trigonometric Identity I
- Trigonometric Identity II

References

Attwood, G. et al. (2017). *Edexcel A level Mathematics - Pure - Year 2*. London: Pearson Education. pp.153-154.